

**Math 115**  
**Spring 2018**  
**Lecture 19**

?  $a^2 + b^2 = c^2$ ?  
 $y = mx + b$ ?  $d = rt$

Class Quiz (Box Your Final Ans.)

① Simplify :  $3(2x + 5y) - 5(x + 3y)$

②  $(-3x^2)^4 \cdot x^7$

③  $\frac{x^{10}y^8}{x^2y^7}$

④  $(2x + 5)(2x - 5)$

## Special Product with binomials

$$\begin{aligned}
 \textcircled{1} \quad (A + B)^2 &= (A+B)(A+B) \\
 &= A^2 + AB + BA + B^2 \\
 &= A^2 + AB + AB + B^2 \\
 &= \boxed{A^2 + 2AB + B^2}
 \end{aligned}$$

ex:

$$\begin{aligned}
 (x+5)^2 &= x^2 + 2(x)(5) + 5^2 \\
 &= x^2 + 10x + 25
 \end{aligned}$$

$$\begin{aligned}
 (3x+4)^2 &= (3x)^2 + 2(3x)(4) + (4)^2 \\
 &= \boxed{9x^2 + 24x + 16}
 \end{aligned}$$

$$\begin{aligned}
 (4x^2 + y^3)^2 &= (4x^2)^2 + 2(4x^2)(y^3) + (y^3)^2 \\
 &= 16x^4 + 8x^2y^3 + y^6
 \end{aligned}$$

Your turn:

$$\textcircled{1} \quad (x + 10)^2 = x^2 + 2(\underline{x})(\underline{10}) + 10^2 = \boxed{x^2 + 20x + 100}$$

$$\begin{aligned}
 \textcircled{2} \quad (4x^3 + x^2)^2 &= (4x^3)^2 + 2(4x^3)(x^2) + (x^2)^2 \\
 &= \boxed{16x^6 + 8x^5 + x^4}
 \end{aligned}$$

$x^m \cdot x^n = x^{m+n}$   
 $(x^m)^n = x^{m \cdot n}$

Special Product with binomials

$$\begin{aligned} \textcircled{a} (A - B)^2 &= (A - B)(A - B) \\ &= A^2 - AB - BA + B^2 \\ &= A^2 - AB - AB + B^2 \\ &= \boxed{A^2 - 2AB + B^2} \end{aligned}$$

$$\begin{aligned} (x - 4)^2 &= x^2 - 2(x)(4) + (4)^2 \\ &= \boxed{x^2 - 8x + 16} \end{aligned}$$

$$\begin{aligned} (5x - 8)^2 &= (5x)^2 - 2(5x)(8) + (8)^2 \\ &= 25x^2 - 80x + 64 \end{aligned}$$

$$\begin{aligned} (2x^5 - 3y^4)^2 &= (2x^5)^2 - 2(2x^5)(3y^4) + (3y^4)^2 \\ &= \boxed{4x^{10} - 12x^5y^4 + 9y^8} \end{aligned}$$

$$(3x^7 - x^4)^2 = (3x^7)^2 - 2(3x^7)(x^4) + (x^4)^2$$

$$= 9x^{14} - 6x^{11} + x^8$$

$$(10x^6 - 7x^4)^2 = (10x^6)^2 - 2(10x^6)(7x^4) + (7x^4)^2$$

$$= 100x^{12} - 140x^6x^4 + 49x^8$$

$$= \boxed{100x^{12} - 140x^{10} + 49x^8}$$

Special Product with binomials

$$\textcircled{3} (A+B)(A-B) = A^2 - AB + BA - B^2$$

Conjugates

$$= A^2 - \cancel{AB} + \cancel{BA} - B^2$$

$$= A^2 - B^2$$

Difference of  
two squares

$$(x+6)(x-6) = (x)^2 - (6)^2$$

Conjugates

Difference of  
two sqrs

$$= \boxed{x^2 - 36}$$

$$\underbrace{(7x^3 + 5)(7x^3 - 5)}_{\text{Conjugates}} = \underbrace{(7x^3)^2 - (5)^2}_{\text{Difference of two sqrs}}$$

$$= 49x^6 - 25$$

$$\underbrace{(4x^8 + 9y^3)(4x^8 - 9y^3)}_{\text{Conjugates}} = \underbrace{(4x^8)^2 - (9y^3)^2}_{\text{Difference of two sqrs}}$$

$$= 16x^{16} - 81y^6$$

$$\underbrace{(12x^7 - 11x^5)(12x^7 + 11x^5)}_{\text{Conjugates}} =$$

$$(12x^7)^2 - (11x^5)^2$$

Difference of two sqrs

$$= 144x^{14} - 121x^{10}$$

Simplify:

$$\textcircled{1} (7.8 \times 10^{13})(9.5 \times 10^{32})$$

$$= 74.1 \times 10^{45}$$

$$= 7.41 \times 10^1 \times 10^{45}$$

$$= 7.41 \times 10^{46}$$

$$\textcircled{2} \frac{8.2 \times 10^{-12}}{4.1 \times 10^{15}}$$

$$= 2 \times 10^{-12-15} = 2 \times 10^{-27}$$

Use exponential rules to Simplify

$$\textcircled{1} x^{-6} = \frac{1}{x^6}$$

$$\textcircled{2} (x^{-4})^5 = x^{-20}$$

$$= \frac{1}{x^{20}}$$

$$\textcircled{3} \frac{x^{-3}y^{10}}{x^2y^{-6}} = \frac{y^{10}y^6}{x^2x^3}$$

$$= \frac{y^{16}}{x^5}$$

$$\textcircled{4} \left( \frac{3x^5}{2y^8} \right)^{-2}$$

$$= \left( \frac{2y^8}{3x^5} \right)^2 = \frac{4y^{16}}{9x^{10}}$$

$$\textcircled{5} \left( \frac{-5x^{-2}}{2y^{-4}} \right)^{-3}$$

$$= \left( \frac{-5y^4}{2x^2} \right)^{-3} = \left( \frac{2x^2}{-5y^4} \right)^3$$

$$= \frac{8x^6}{-125y^{12}}$$

$$= \boxed{\frac{-8x^6}{125y^{12}}}$$

⑥ Give Degree & Coef. of each term:

$$24xy^3 - 100x^6y^2 + xy - 400$$

$$D = 1+3=4$$

$$C = 24$$

$$D = 6+2=8$$

$$C = -100$$

$$D = 1+1=2$$

$$C = 1$$

$$\text{Constant} \\ D = 0$$

$$D = 8$$

$$L.C. = -100$$

Polynomial

Monomial

$$\frac{15x^2 - 10x}{5x} = \frac{15x^2}{5x} - \frac{10x}{5x}$$

$$= \boxed{3x - 2}$$

$$\frac{35x^7 - 14x^3 + 7x}{7x} = \frac{35x^7}{7x} - \frac{14x^3}{7x} + \frac{7x}{7x}$$

$$= \boxed{5x^6 - 2x^2 + 1}$$

Divide:  $\frac{45x^6 - 25x^4 + 10x^2}{-5x^2}$

$$= \frac{45x^6}{-5x^2} - \frac{25x^4}{-5x^2} + \frac{10x^2}{-5x^2}$$

$$= \boxed{-9x^4 + 5x^2 - 2}$$

Trinomial

$$D=4$$

$$L.C. = -9$$

$$\text{Const.} = -2$$

Polynomial  
Binomial

Long Division

$$\frac{x^2 - 8x + 15}{x - 3}$$

$$x \boxed{x} = x^2$$

$$x \boxed{-5} = -5x$$

$$\boxed{x - 5}$$

$$\begin{array}{r} x-3 \overline{) x^2 - 8x + 15} \\ \underline{-(x^2 - 3x)} \phantom{+15} \\ -5x \phantom{+15} \\ \underline{-(-5x + 15)} \\ \text{Remainder} \rightarrow 0 \end{array}$$



Divide:  $\frac{x^3 + 5x^2 - 3x + 10}{x + 2}$

$x \boxed{x^2} = x^3$   
 $x \boxed{3x} = 3x^2$   
 $x \boxed{-9} = -9x$

$$\begin{array}{r}
 x^2 + 3x - 9 \\
 x+2 \overline{) x^3 + 5x^2 - 3x + 10} \\
 \underline{-(x^3 + 2x^2)} \phantom{+ 10} \\
 3x^2 - 3x + 10 \\
 \underline{-(3x^2 + 6x)} \phantom{+ 10} \\
 -9x + 10 \\
 \underline{-(-9x - 18)} \\
 \text{Remainder} \rightarrow 28
 \end{array}$$

$x^2 + 3x - 9 + \frac{28}{x+2}$   
 Always

Divide:  $\frac{x^3 - 3x^2 + 4}{x - 2}$

$x \boxed{x^2} = x^3$   
 $x \boxed{-x} = -x^2$   
 $x \boxed{-2} = -2x$

$$\begin{array}{r}
 x^2 - x - 2 \\
 x-2 \overline{) x^3 - 3x^2 + 0x + 4} \\
 \underline{-(x^3 - 2x^2)} \phantom{+ 4} \\
 -x^2 + 0x + 4 \\
 \underline{-(-x^2 + 2x)} \phantom{+ 4} \\
 -2x + 4 \\
 \underline{-(-2x + 4)} \\
 \text{Remainder} \rightarrow 0
 \end{array}$$

$x^2 - x - 2$

Divide: 
$$\frac{14x - 2 - 17x^2 + 6x^3}{2x - 3}$$

$$= \frac{6x^3 - 17x^2 + 14x - 2}{2x - 3}$$

$2x \boxed{3x^2} = 6x^3$   
 $2x \boxed{-4x} = -8x^2$   
 $2x \boxed{1} = 2x$

$$\begin{array}{r}
 3x^2 - 4x + 1 \\
 2x-3 \overline{) 6x^3 - 17x^2 + 14x - 2} \\
 \underline{-(6x^3 - 9x^2)} \phantom{+ 14x - 2} \\
 -8x^2 + 14x - 2 \\
 \underline{-(-8x^2 + 12x)} \phantom{- 2} \\
 2x - 2 \\
 \underline{-(2x - 3)} \\
 \text{Rem.} \rightarrow 1
 \end{array}$$

$$3x^2 - 4x + 1 + \frac{1}{2x-3}$$